Table showing the Expectation of Life (continued).

Age.	By Carlisle Table of Mortality.	By the proposed Hypotheses.	Age.	By Carlisle Table of Mortality.	By the proposed Hypotheses.
	Years. Parts.	Years. Parts.		Years. Parts.	Years. Parts.
71	8·65	8·83	81	5·21	5·50
72	8·16	8·33	82	4·93	5·25
73	7·72	7·83	83	4·63	5·00
74	7·33	7·33	84	4·39	4·75
75	7·01	7·00	85	4·12	4·50
76	6·69	6·75	86	3·90	4·25
77	6·40	6·50	87	3·71	4·00
78	6·12	6·25	88	3·59	3·75
79	5·80	6·00	89	3·47	3·50
80	5·51	5·75	90	3·28	3·25

On a Method of Testing the Solvency of an Assurance Company, with some considerations respecting the Terms upon which a Society's Business may be Purchased. By SAMUEL YOUNGER, *of the Engineers' Life Office.*

IT appears to be an admitted principle in the commercial world, that great evils have a tendency to work their own cure.   Over-trading will bring about a state of things by which eventually it must be checked, and excessive speculation has its limit by the same invariable law.

More than two years ago, it was quite evident that the rapid manner in which new Companies were being formed for the pur-pose of assuring lives would ere long produce sad and humiliating results.   Such an undue multiplication of Offices would of neces-sity render business for any one of them the more difficult to be secured ; and therefore, if obtained, it could but be at a corre-spondingly larger cost.   Competition in this respect has, indeed, been carried to so great an extent, that many of the new Offices have been brought to a complete stand; the mainspring of ulti-mate success—namely, the investment of a certain proportion of the premiums received—was of necessity broken ; and to get business, however evanescent its character might prove to be, not only has it often happened that the whole of the premiums have been expended, but in some instances all the share capital too, and the Office has run largely into debt besides.   At such a crisis, the expedient of amalgamating with another Company, or of selling

its business, has been resorted to; and the possibility of taking this step (often without loss) sufficiently explains why a Society, although going rapidly from bad to worse, should be tempted to prolong its existence even after all reasonable hope of success had disappeared.

The terms upon which transfers of this kind are made between one young Company and another are frequently of the most remarkable description. A strange notion appears to prevail amongst many of them—that for a life business of seven, eight, or even ten years' standing, a year's purchase may safely be given; and that for one four or five years old, a year and a half or two years' purchase is not excessive. The actual process is beautifully simple. The business of one Office, A, established say five years, is taken by another Office, B (the latter being perhaps in anything but a flourishing condition); and, as the purchase money for such business, B takes the whole of A's debts, or such portion of them as will absorb in their payment the entire premiums receivable upon A's policies for a year and a half or two years, supposing none of them to lapse. The transaction is based upon no correct principle or calculation whatever, the terms above mentioned being regarded as axiomatic truths. A feverish anxiety is manifested to grasp at an increase of income, without a due consideration of the price which it will *ultimately* cost. The tacit assumption is generally made, that none of the policies will be forfeited: this point, however, should not be disposed of so summarily, it evidently claims particular attention; for even when the business of an Office is transferred to another of longer standing, a considerable number of policies will in all probability be abandoned before a year has elapsed—that is to say, many of the assured whose policies were in force at the time of the transfer will not pay a single premium to the purchasing Company. Thus not only will the anticipated income suffer diminution, but the latter Office will be open to the risk upon these policies for a certain period, without receiving any compensation for the liability. It would be easy to account for the many lapses, by considering the shock which persons connected with these institutions must sustain when they find the Society in which their savings have been invested unable to continue business: how are they to know but that the Office to which their policies have been transferred will be the next to fail? It is, however, unnecessary for my present purpose to enter into this subject: one thing at least is certain—that a considerable allowance should be made for the contingency alluded to, in all valuations having reference to

the purchase of a business. These observations, it must be remembered, refer to the case of a young Company purchasing another of tender years; but it can hardly be supposed they would apply to some transfers we have seen lately, for in these instances all *à priori* reasoning must lead to a totally opposite conclusion.

In justification of the excessive prices to which allusion has been made, a comparison is frequently instituted between the annual income which the business about to be purchased will produce, and the sum which such an income would cost a recently established Society to obtain. Such comparisons, it is plain, can have no bearing whatever upon the matter; the results to which they lead sufficiently condemn them, for neither this nor any other argument can be sound which proves that a greater sum may be given for a business than a careful valuation shows it to be worth.

The question now arises, as to the mode of calculation it is expedient to follow, in order to arrive at a safe estimate of the terms upon which a transfer may be made. Now there are many objections to the method sometimes adopted, of regulating these terms solely by the difference in present value of the future premiums and the sums assured; for, to say nothing of the disturbing effect upon such computations of the forfeiture of many of the policies, there is a large element of uncertainty in the amount that will be required for bonuses, should a considerable portion of the business be on the participating scale. These considerations seem to point to the necessity of looking rather at what the position of the selling Company would have been at the present time, if its expenditure had been kept within proper bounds and limited to the marginal additions to the net premiums. It will perhaps be urged that this is too severe a standard, inasmuch as no allowance is made for preliminary expenses, which are always very heavy; but what has the purchasing Company to do with such an item? one Society cannot be expected to make a bad bargain in order to rid another of its liabilities. The loss must, of course, fall upon the unsuccessful Office.

The following method of determining whether a Company is in a solvent state will, I think, be found serviceable in guiding an actuary to the proper terms of a transfer:—

Suppose an Office to have been established n years; and let $a_1, a_2, a_3 \ldots \ldots a_n$ be respectively the gross annual income from premiums in the first, second, third, &c. years of its existence, and $\delta_1, \delta_2, \delta_3 \ldots \ldots \delta_n$ the claims in the several years. Then,

if the premiums are loaded $\left(\dfrac{1}{k}\right)$th part—that is, if, taking one with another, each Office premium $=\left(1+\dfrac{1}{k}\right)\times$ net premium—it follows that the portion of the first year's income which should in strictness have been invested at the end of that year is

$$\frac{(k-1)a_1}{k}-\delta_1;$$

an additional investment of

$$\frac{(k-1)a_2}{k}-\delta_2$$

being made at the end of the second year, and so on. Thus the funds at the termination of the nth year should amount to

$$f_n=\left\{\frac{(k-1)a_1}{k}-\delta_1\right\}\mu^{n-1}+\left\{\frac{(k-1)a_2}{k}-\delta_2\right\}\mu^{n-2}\ \ldots\ \ldots\ \frac{(k-1)a_n}{k}-\delta_n,$$

where $\mu=1+i$.

The funds here alluded to are those necessary to provide for the sums originally assured under existing policies; but if any bonuses have been allotted, the present value of the outstanding additions must be added to f_n; also the paid-up capital (if any) of the Company, and the value of existing annuities.

Denoting the values of the annuities and bonuses by A and B respectively, and the share capital by C, the total invested funds of the Company should be

$$f_n+\text{A}+\text{B}+\text{C}\ \ .\ \ \ .\ \ \ .\ \ \ .\ \ \ .\ \ (1);$$

if they fall short of this amount, the Office cannot, strictly speaking, be pronounced solvent at the time. But there is little doubt, even if a large deficiency were found to exist, that by a prudent and economical system of management the sum overspent might be replaced by degrees.

Should it happen that no portion of the deficiency at the end of the nth year is made up during the year following, the position of the Company would be worse at the end of the $(n+1)^{\text{th}}$ year than at the beginning, by at least the twelve months' interest on the sum which its funds were short when the year commenced; and this too, if the amount represented by $\dfrac{(k-1)a_{n+1}}{k}$ had not been encroached upon but for the payment of annuities and policy claims. We have here a fact which, although obvious enough to any actuary, cannot be too strongly pressed upon the attention of directors—that, notwithstanding what may seem a large sum may be laid by in a single year, it by no means follows that the Society has made any progress towards a healthy and substantial con-

dition; but, on the contrary, it may still be not only in an insolvent but also in a retrograding state.

When the purchase of a business is contemplated and no payment is to be made by the selling Company, it must be a matter of grave consideration for the actuary, whether the margin on the premiums to be received will be sufficient to replace the greater part of the heavy sum indicated by (1), with all its accumulations of compound interest; leaving enough to meet unavoidable expenses, such as commission, &c., and to provide for future bonuses besides. Any actual gain to the purchasing Company in a matter of this kind must be looked for in the extended connection which the transaction brings, it being tolerably certain that none can ever be expected from the business bought.

It will be seen by the mode of calculation adopted, that f_n includes the $\left(\dfrac{k-1}{k}\right)$th part of all premiums received upon lapsed policies. This is a somewhat larger proportion of such premiums than it is necessary to keep invested after the risks have ceased; but the excess will not amount to much, particularly in the case of a young Company.

It certainly becomes a question, whether a Life Office may not spend the whole of the premiums on forfeited policies without in any way injuring its stability; for, although the principle is antagonistic to theory, we may presume the number of cases in which policies are continued until the happening of the events upon which they become claims, will in the long run be sufficiently great to warrant the assumption that, among these alone, chance will have room enough to play. This hypothesis is, in fact, adopted when the amount of divisible surplus is founded upon a valuation which takes into account the difference merely between the present value of the future premiums and the sums assured on policies in existence at the time. If the principle be admitted, the amount f_n will be considerably diminished.

Whether insuperable difficulties would arise in applying the preceding test of solvency to a Company of long standing, I am not able to say; but in the case of a young Office no obstacle whatever will be found. What such a Society's position *ought to be*, can, by the method suggested, be most readily ascertained; and this, compared with what *it is*, will point out the limit within which the future expenditure should be kept in order that the undertaking may gradually advance, and in course of time be crowned with a fair measure of success.